Wave Interference 2 (problem solving session) Lesson Plan
GK-12 Program – Harvard University
October 14, 2003

## Review of Basic Wave Ideas:

1. Examples of waves: water waves, stadium waves, sound waves

2. A wave is a disturbance that travels through a medium.

3. There are two types of waves: transverse and longitudinal.

    a. A transverse wave is a wave in which the motion of the medium is perpendicular to the direction it is traveling (for example waves on a string).

    b. A longitudinal wave is a wave in which the motion of the medium is parallel to the direction that the wave is traveling (for example the pushing and pulling of a slinky, or sound waves).

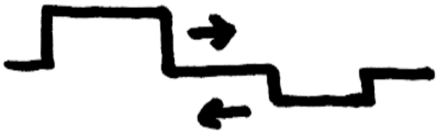
We will talk about interference and standing waves using transverse waves as examples (since they are easier to draw). It is important to remember, however, that the same is true about longitudinal waves.

## Interference Between Wave Pulses:

1. To get the net wave produced when two waves interfere, simply add (or subtract) the two. See the two examples below:

during interference

a.



b.

2. To calculate the time when two pulses will overlap (see problem 1), you will need to know the velocity of each wave and the distance between them. Find out at what location the two waves overlap. Then, calculate how long it will take for one of the waves to reach that point. This can be obtained using the following relationship:

$$velocity = \frac{distance}{time}$$

*(Homework Hint: The time it takes for two pulses to hit which are each traveling at 1m/s towards each other is the same time it would take one pulse traveling at 2m/s towards a pulse at rest. Why is this?)*

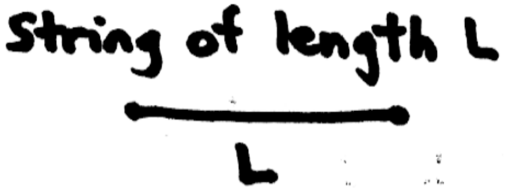
Standing Waves:
(see section 13-11 of Giancoli)

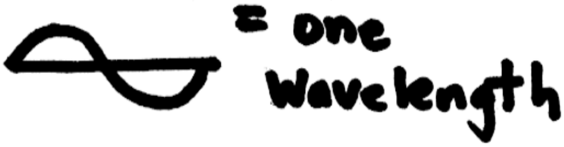
1. Below is a picture of a standing wave. Nodes are points of no vibration. Antinodes are points of maximum vibration.

2. Examples of standing waves: vibrating guitar string, pressure waves in a clarinet

3. Standing waves are not unrelated to traveling waves. Two opposite moving traveling waves can superimpose to form a standing wave. Consider a wave traveling down a rope, which has one end attached to a pole. When the wave hits the pole, the wave will be reflected backwards and travel in the opposite direction. The reflected wave will interfere with the incident wave. If the rope is vibrating at a resonant frequency, this interference will produce a standing wave.

Resonant Frequencies of a String Fixed at Both Ends:





1.

a. Fundamental or first harmonic. This is the largest wavelength that will fit on the string.

$L = \frac{1}{2}\lambda_1$

b. First overtone or second harmonic:

$L = \lambda_2$

c. Second overtone or third harmonic:

$L = \frac{3\lambda_3}{2}$

2. We see that there is a pattern! This is:

$$L = \frac{n * \lambda_n}{2} \qquad \text{where n = 1, 2, 3, …}$$

The integer n labels the number of the harmonic; n = 1 for the fundamental, n = 2 corresponds to the second harmonic, and so on.

We can rearrange the above equation to solve for $\lambda_n$:

$$\lambda_n = \frac{2 * L}{n} \qquad \text{where n = 1, 2, 3, …}$$

3. The wavelength and frequency of the wave are related by the following equation:

$$f_n = \frac{v}{\lambda_n}$$

v is the velocity of the wave on the string. This will depend on the tension and the mass density of the string.

Plugging in for the wavelength, we get:

$$f_n = \frac{v * n}{2 * L} \qquad \text{where n = 1, 2, 3, …}$$

Thus, we see that the second harmonic has a frequency that is twice that of the fundamental, etc…

## Resonant Frequencies of a String Fixed at One End and Free at the Other:

1. Now, the ends of the string have different boundary conditions than before. It is hard to imagine setting this up on a string, but remember that we said we will draw waves on strings (transverse waves) to make illustrations since they are easier to draw but that the same result applies to longitudinal waves? These boundary conditions often occur with pressure waves in a tube. The standing waves in a flute, for example, have these boundary conditions.

2. Below are pictures of the three largest wavelengths that fit onto the string:

a. Fundamental or first harmonic. This is the largest wavelength that will fit on the string.

$L=\frac{1}{4}\lambda_1$

b. First overtone or second harmonic:

c. Second overtone or third harmonic:

<u>Summary:</u> Waves traveling on a string of fixed length interfere with waves that have reflected off the end and are traveling back in the opposite direction. At certain frequencies, standing waves can be produced. These frequencies are called the resonant frequencies. Points of no vibration are called nodes, and points of maximum vibration are called antinodes. Since the string is at rest at the nodes, no energy flows past these points and hence energy is not transmitted down the string. A standing wave does appear to be standing still!